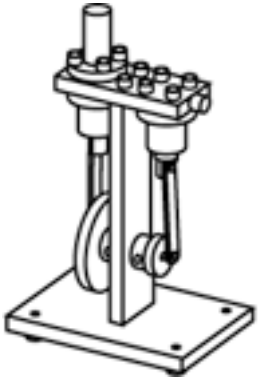
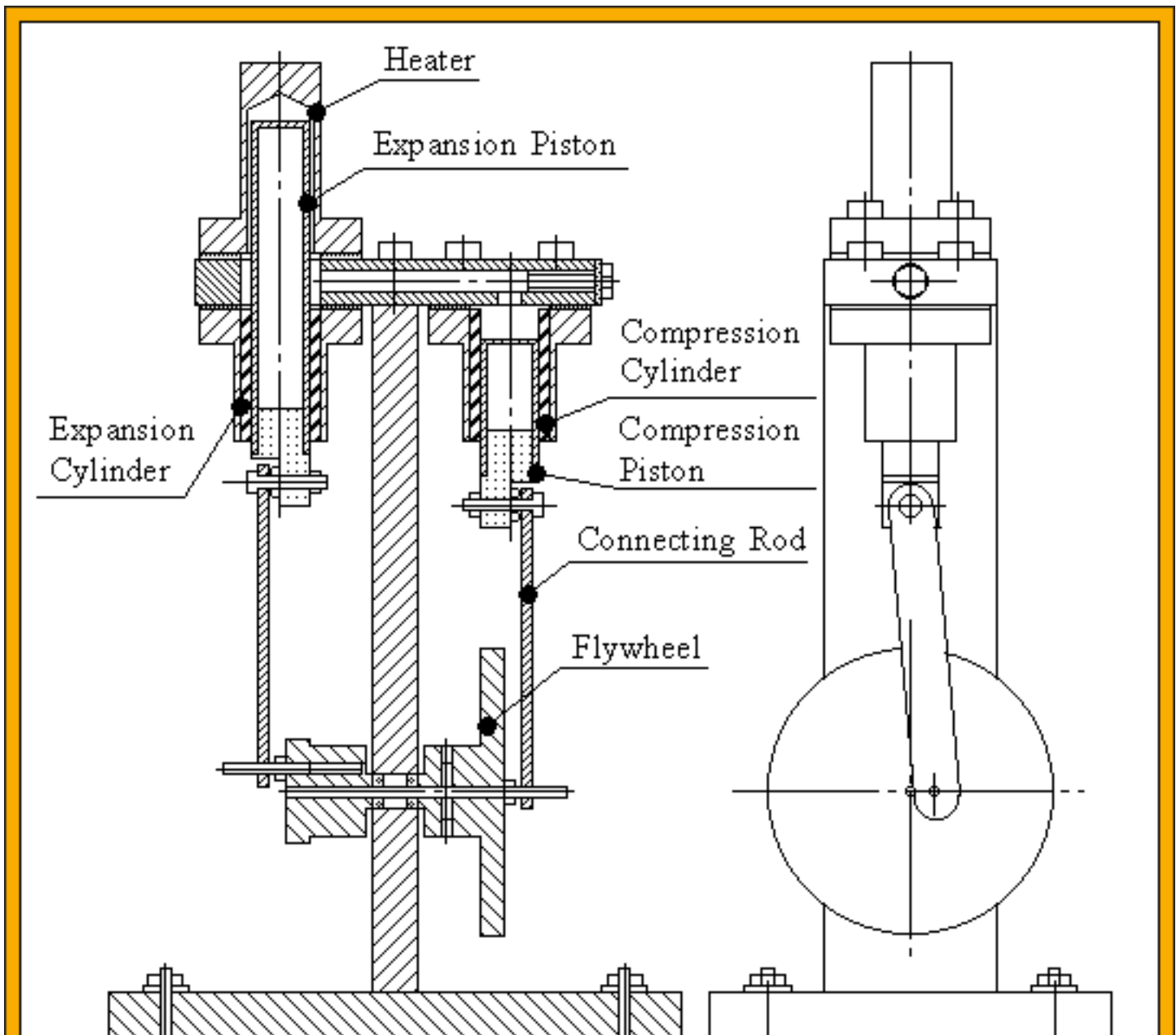


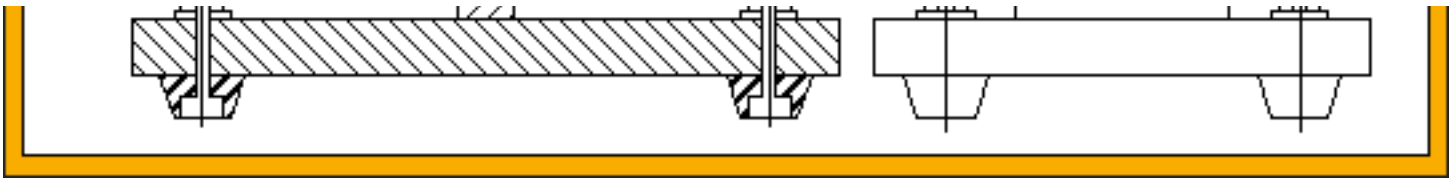
Building of Model Stirling Engine

Plans of the model Stirling engine



[Japanese page](#)





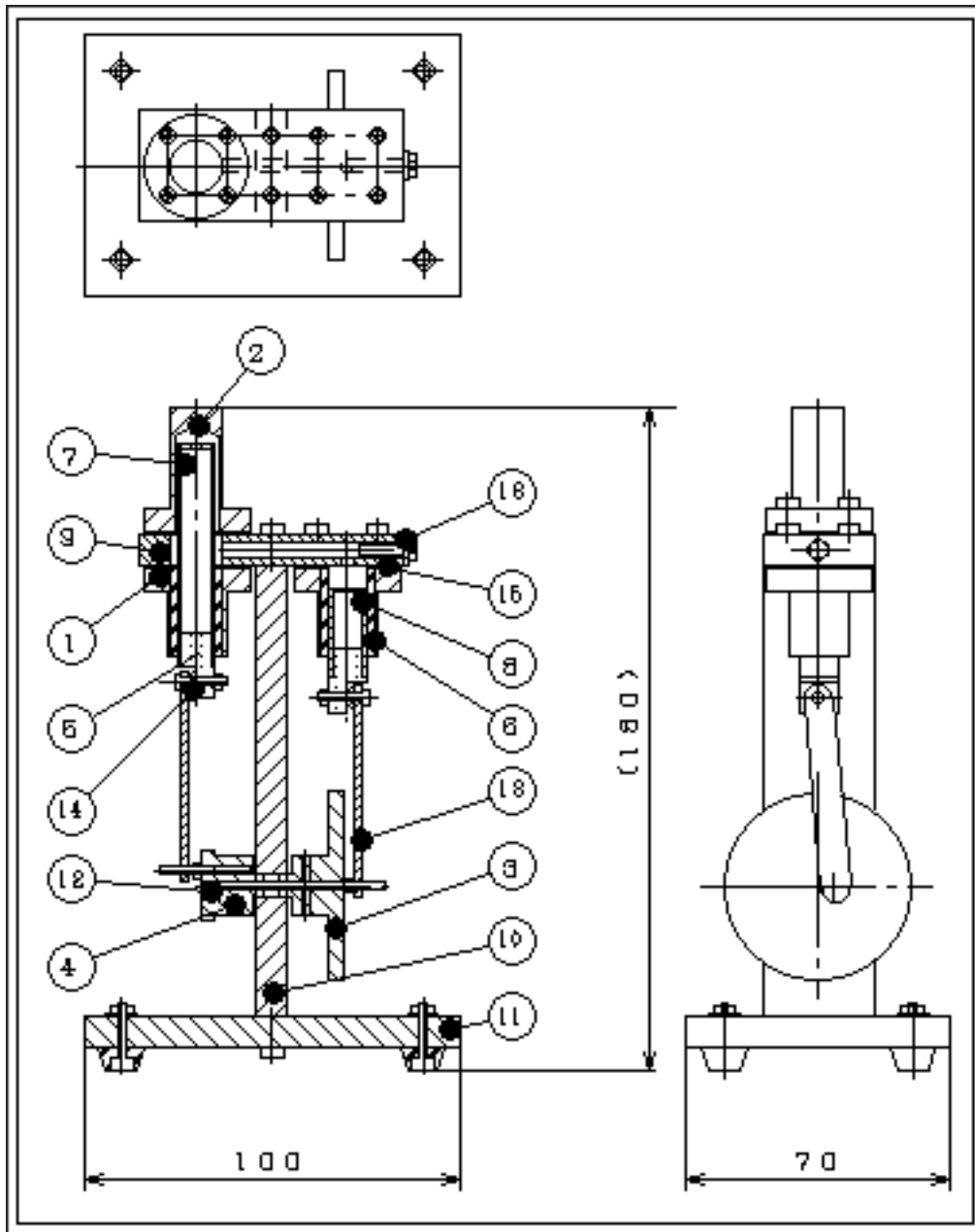
Follows are the plans of the model Stirling engine, which were delivered to the students.

- [Assembly drawing of the model Stirling engine \(1\)](#)
- [Assembly drawing of the model Stirling engine \(2\)](#)
- [Suggestions to Assemble the model Stirling engine](#)



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Assembling drawing - No.1



Number	Name	Material	the number of parts	Notes
1	Cylinder cover	aluminum	2	
2	Heater	stainless	1	
3	Flywheel	brass	1	
4	Crank disk	brass	1	
5	Piston holder	aluminum	2	
6	Cylinder	glass	2	Medical syringe
7	Hot piston	glass	1	Medical syringe
8	Cold piston	glass	1	Medical syringe
9	Joint board	aluminum	1	
10	Frame	aluminum	1	
11	Base	aluminum	1	
12	Shaft	stainless	1	
13	Connecting rod	aluminum	2	
14	Bush	steel wire	2	
15	Gasket	silicone rubber	3	
16	Gasket	silicone rubber	1	

[1: Cylinder Cover](#) [2: Heater](#) [3: Flywheel](#) [4: Crank Disk](#) [5: Piston Holder](#)

[6: Cylinder](#) [7: Hot Piston](#) [8: Cold Piston](#) [9: Joint Board](#) [10: Frame](#) [11: Base](#)

[12: Shaft](#) [13: Connecting Rod](#) [14: Bush](#) [15: Gasket](#) [16: Gasket](#)

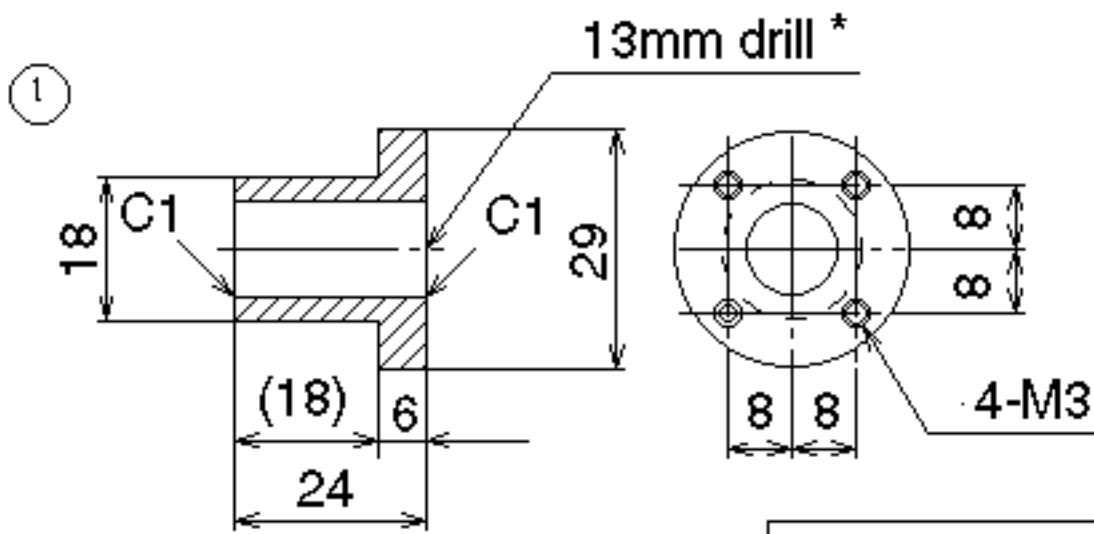


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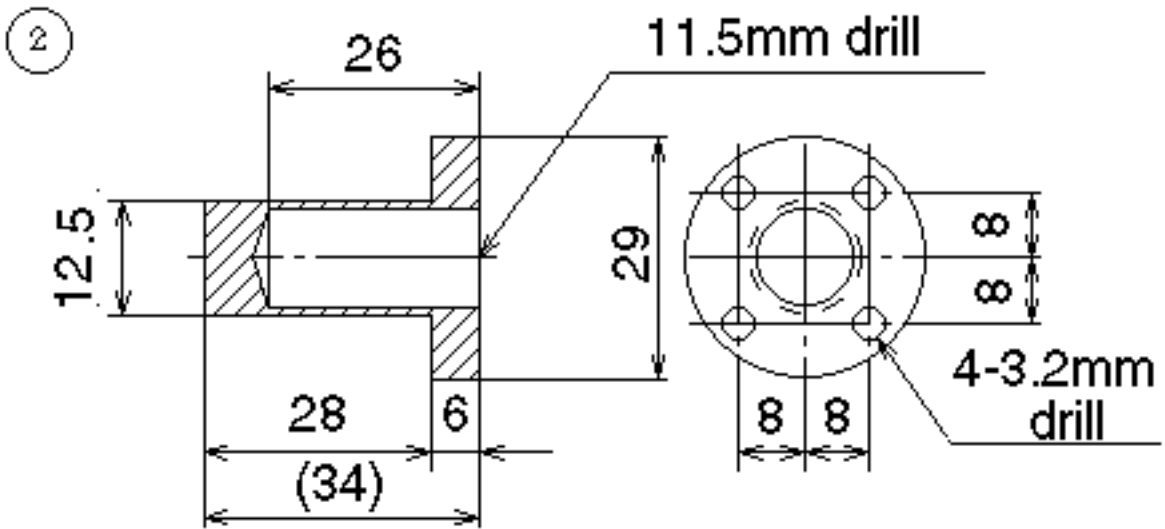
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Number	Name	Material	Number of parts	Notes
1	Cylinder cover	aluminum	2	



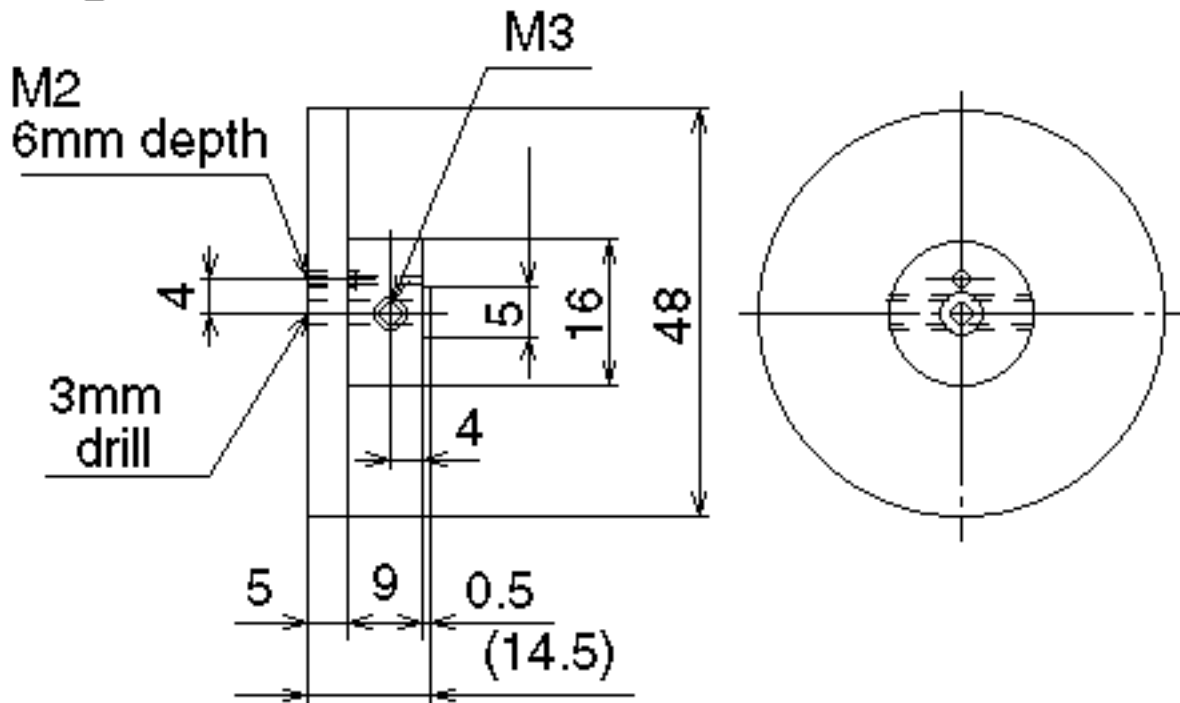
* Note:
Fit to diameter of
medical syringes.

Number	Name	Material	Number of parts	Notes
2	Heater	stainless	1	



Number	Name	Material	Number of parts	Notes
3	Flywheel	brass	1	

3

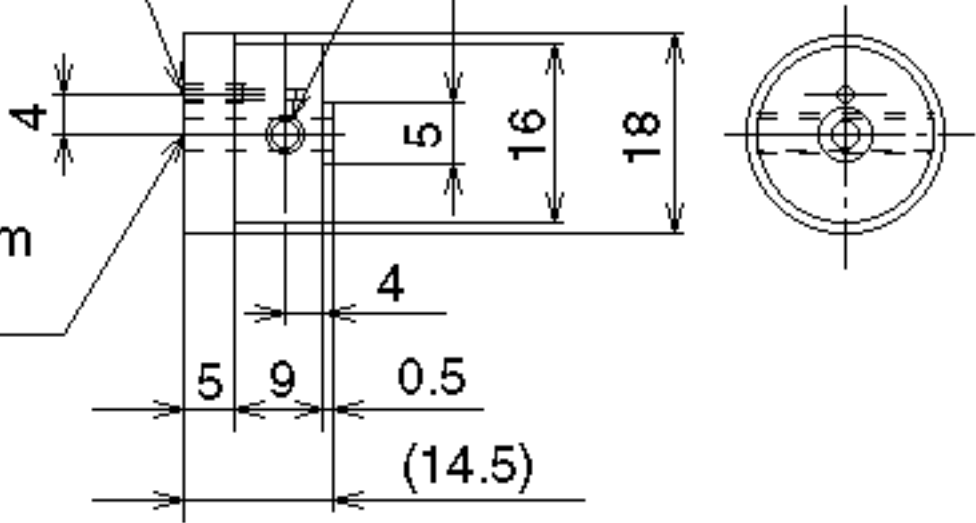


4

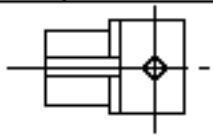
Number	Name	Material	Number of parts	Notes
4	Crank disk	brass	1	

M2,
6mm depth

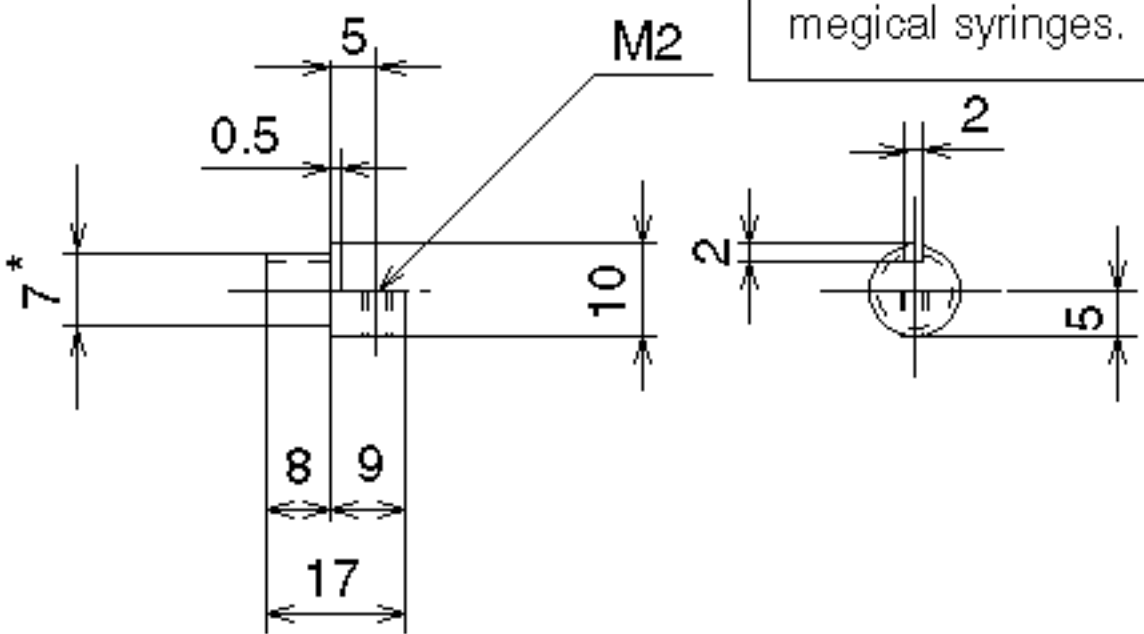
M3



Number	Name	Material	Number of parts	Notes
5	Piston holder	aluminum	2	

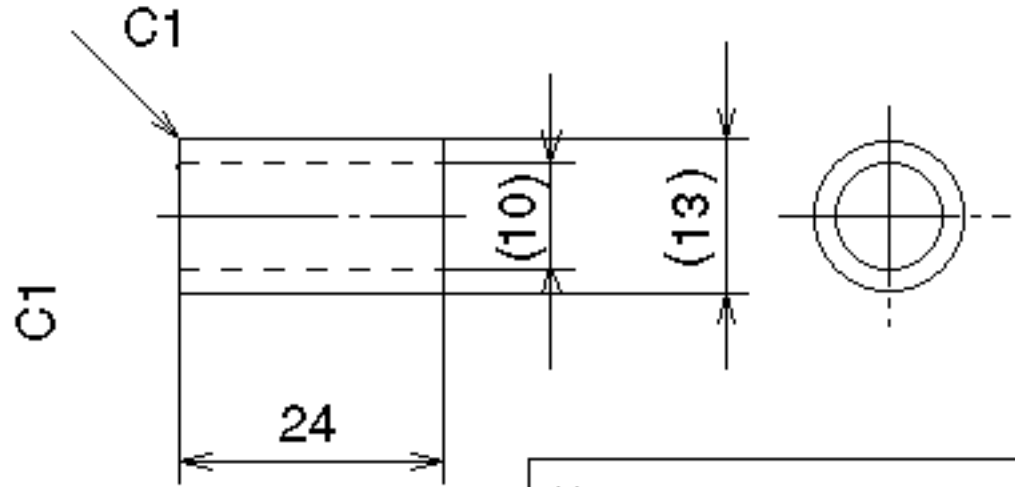


* Note:
Fit to diameter of
medical syringes.



6

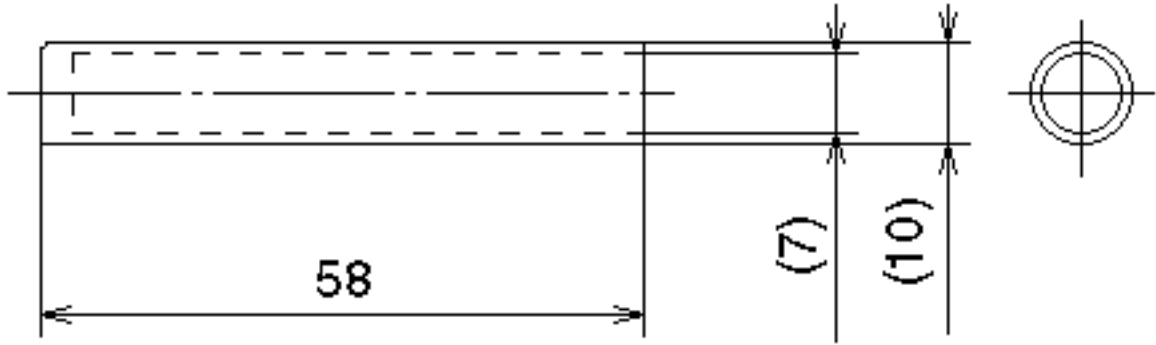
Number	Name	Material	Number of parts	Notes
6	Cylinder	glass	2	Medical syringe



Note:
Use 3cc medical syringes.

Number	Name	Material	Number of parts	Notes
7	Hot piston	glass	1	Medical syringe

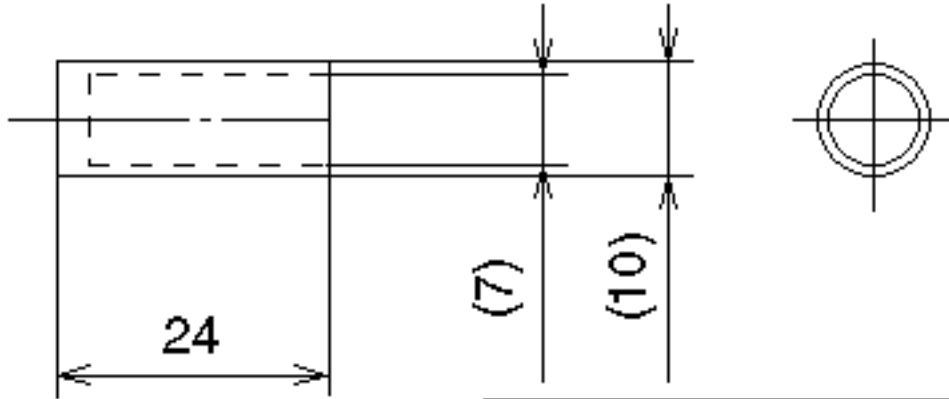
7



Note:
Use 3cc medical syringes.

Number	Name	Material	Number of parts	Notes
8	Cold piston	glass	1	Medical syringe

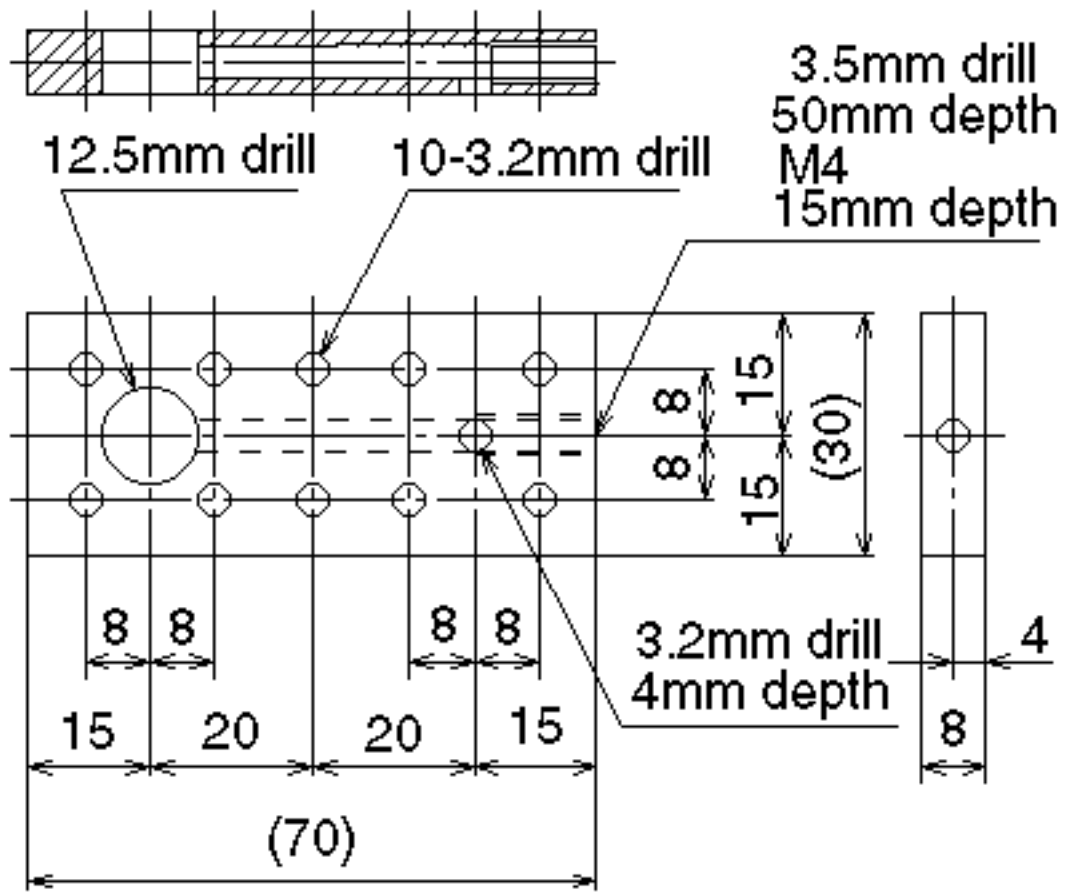
8



Note:
Use 3cc medical syringes.

9

Number	Name	Material	Number of parts	Notes
9	Joint board	aluminum	1	

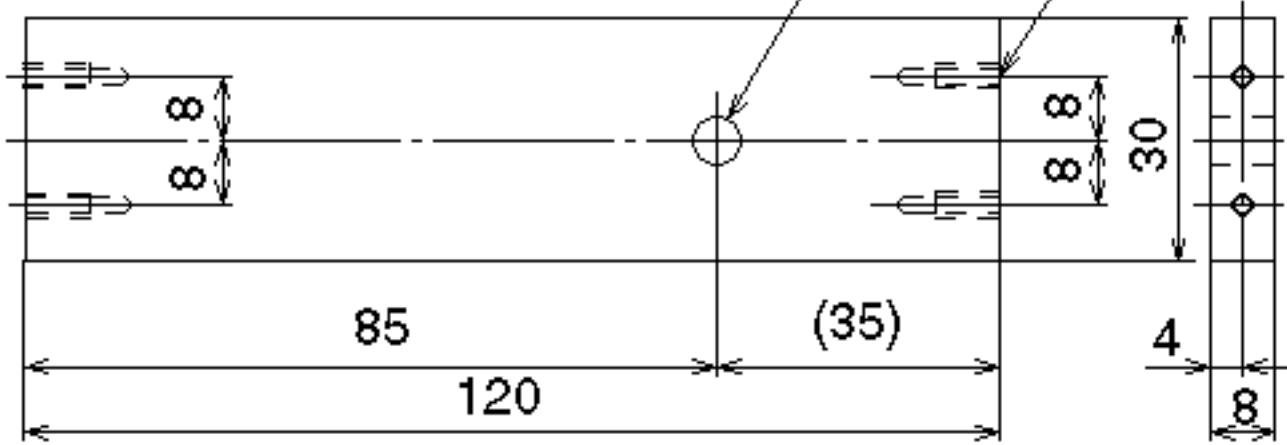


Number	Name	Material	Number of parts	Notes
10	Frame	aluminum	1	

10

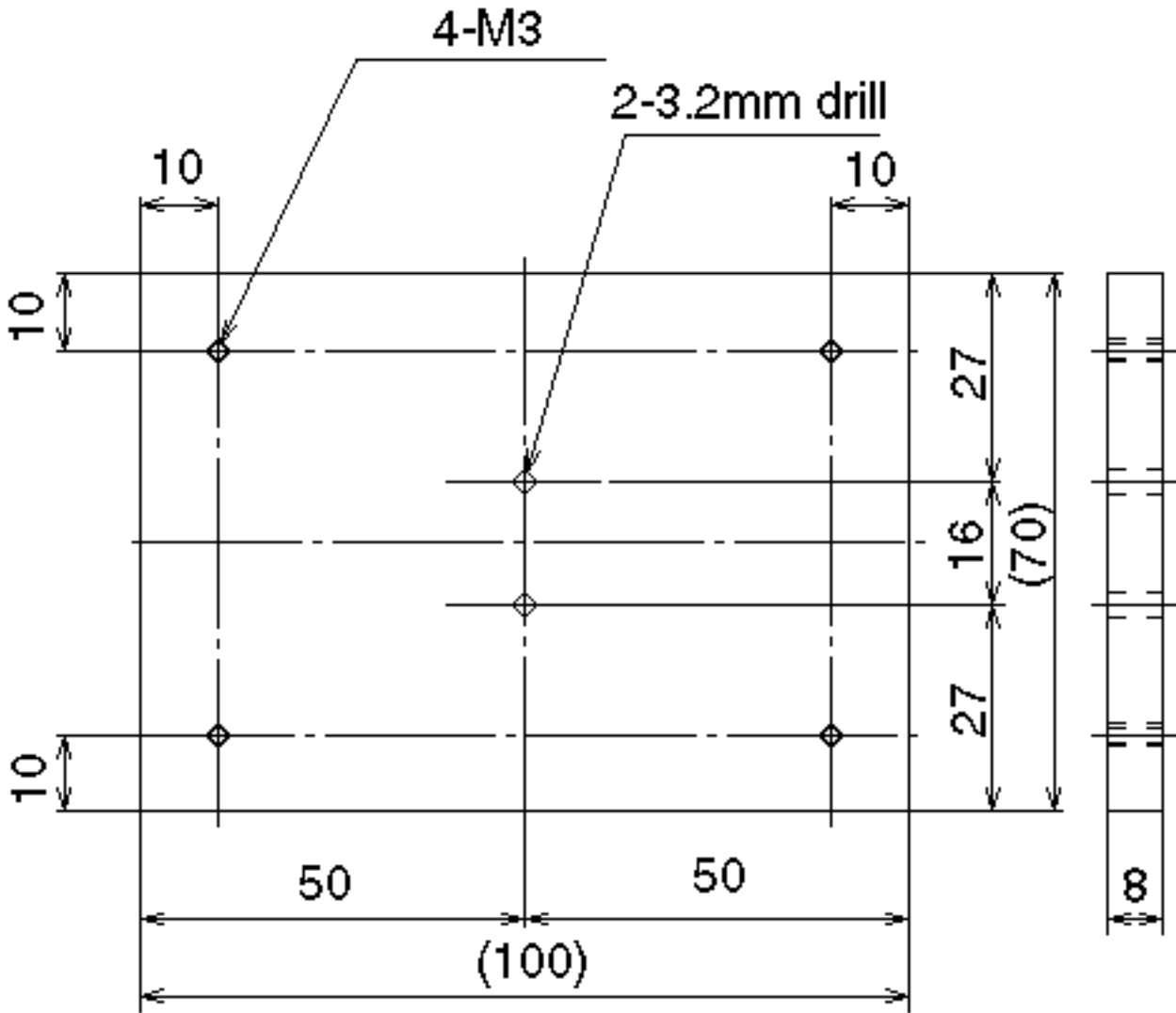
4-M3, 8mm depth

6mm drill



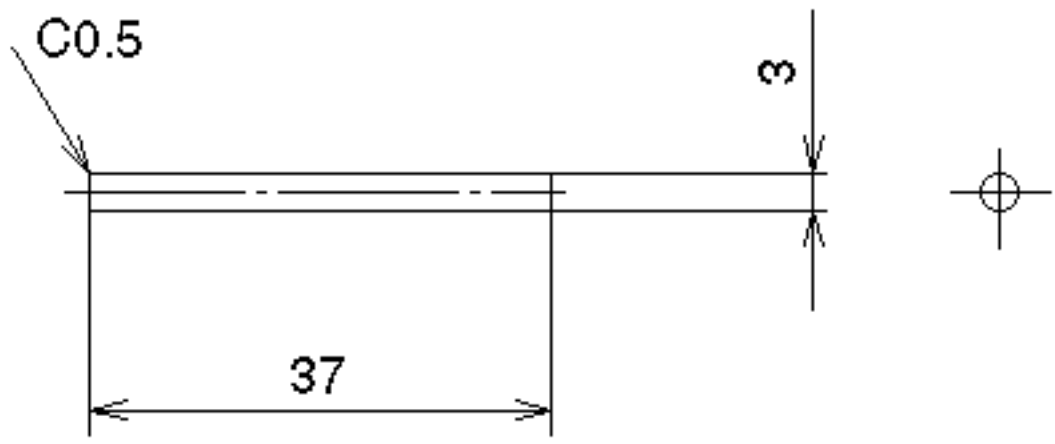
Number	Name	Material	Number of parts	Notes
11	Base	aluminum	1	

11



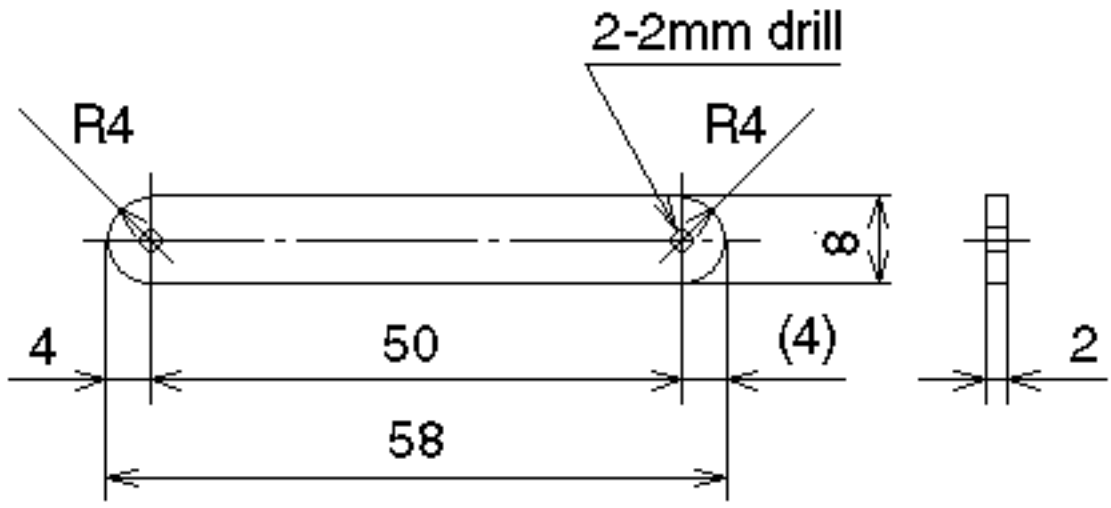
Number	Name	Material	Number of parts	Notes
12	Shaft	stainless	1	

12



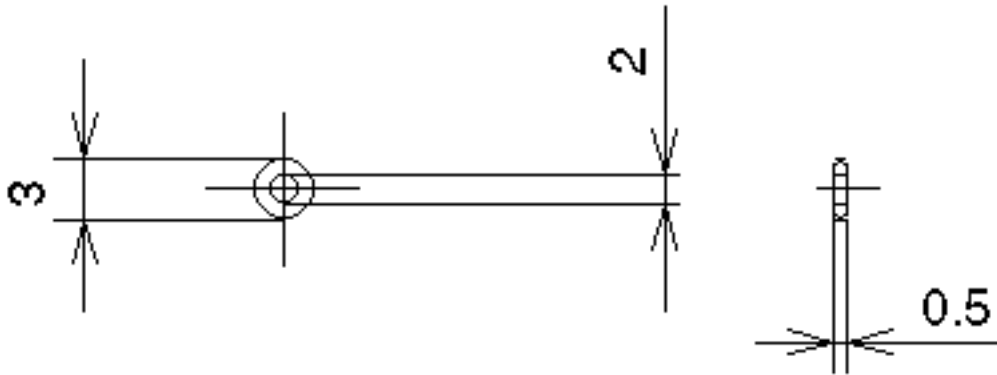
Number	Name	Material	Number of parts	Notes
13	Connecting rod	aluminum	2	

13



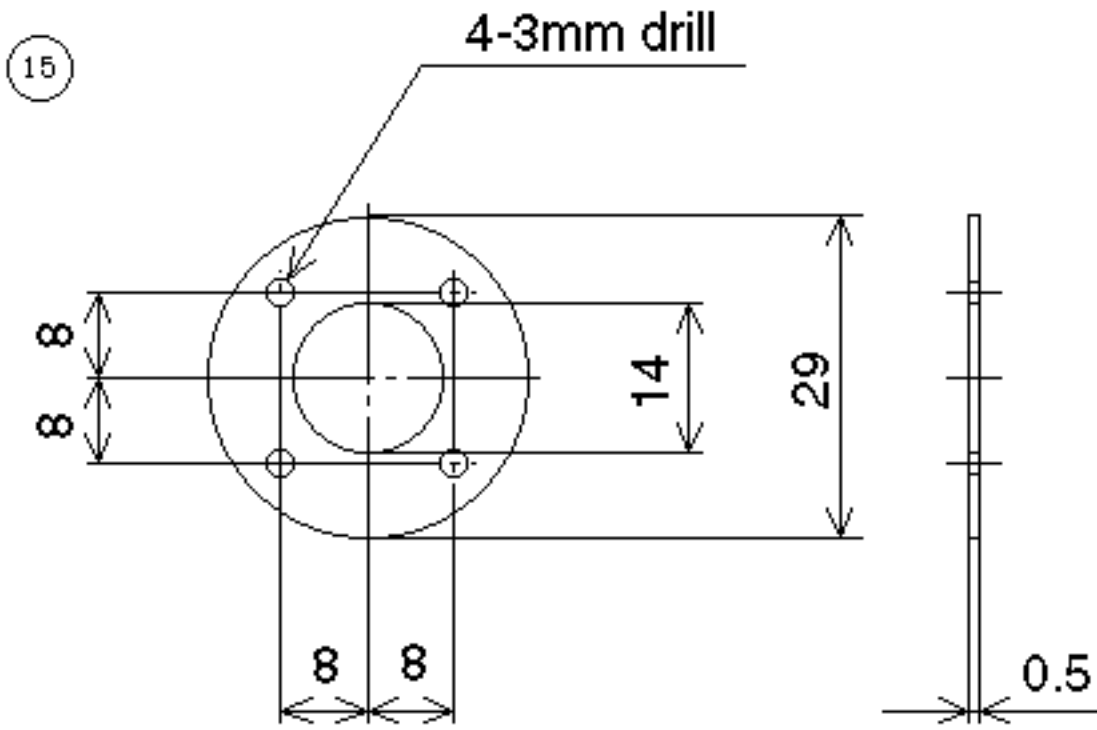
14

Number	Name	Material	Number of parts	Notes
14	Bush	steel wire	2	



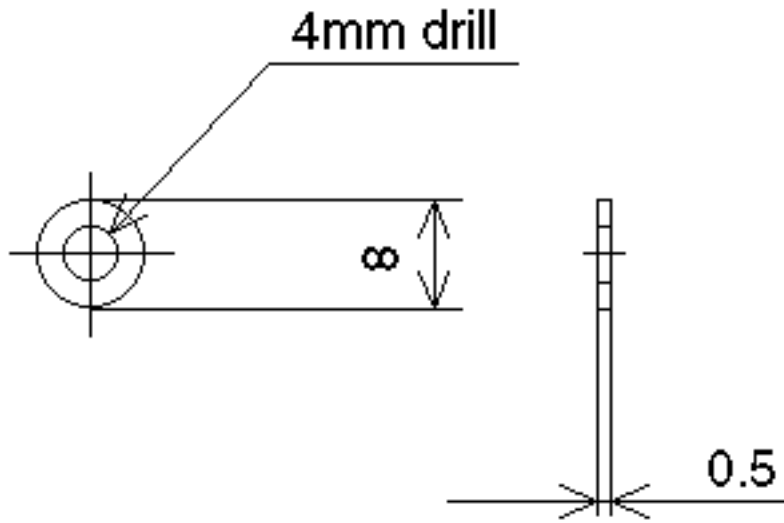
Note:
Bend a steel wire (0.5mm diameter).

Number	Name	Material	Number of parts	Notes
15	Gasket	silicone rubber	3	

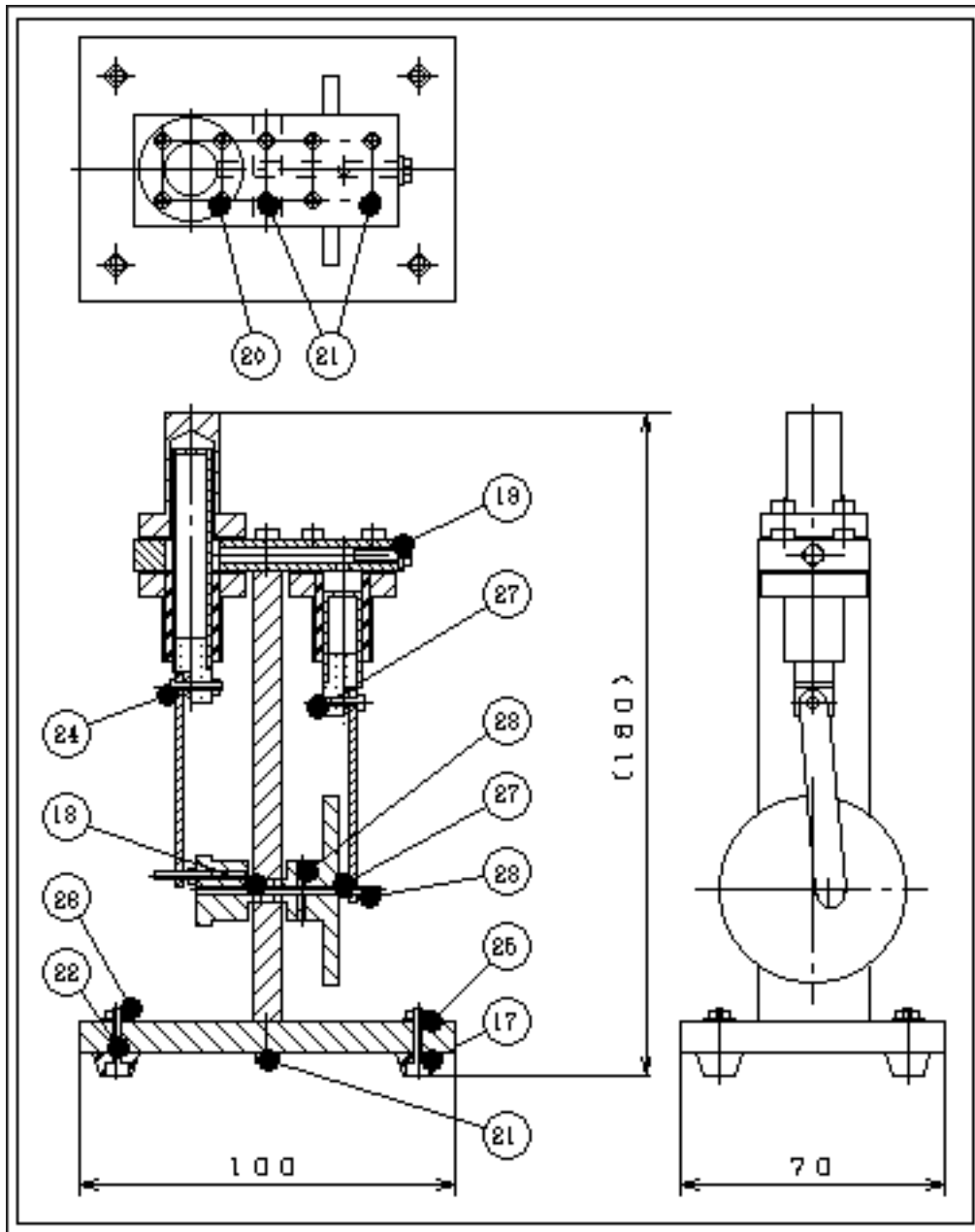


Number	Name	Material	Number of parts	Notes
16	Gasket	silicone rubber	1	

18



Assembling drawing - No.2



Number	Name	Material	the number of parts	Notes
17	Leg	rubber	4	
18	Bearing	steel	2	MF63 (Outer 6mm, Inner 3mm)
19	Bolt	steel	1	M4 x 12mm
20	Bolt	steel	4	M3 x 20mm
21	Bolt	steel	8	M3 x 12mm
22	Bolt	steel	4	M3 x 15mm
23	Bolt	steel	2	M2 x 20mm
24	Bolt	steel	2	M2 x 10mm
25	Washer	steel	4	M3
26	Nut	steel	4	M3
27	Nut	steel	4	M2
28	Cap screw	steel	4	M3 x 5mm

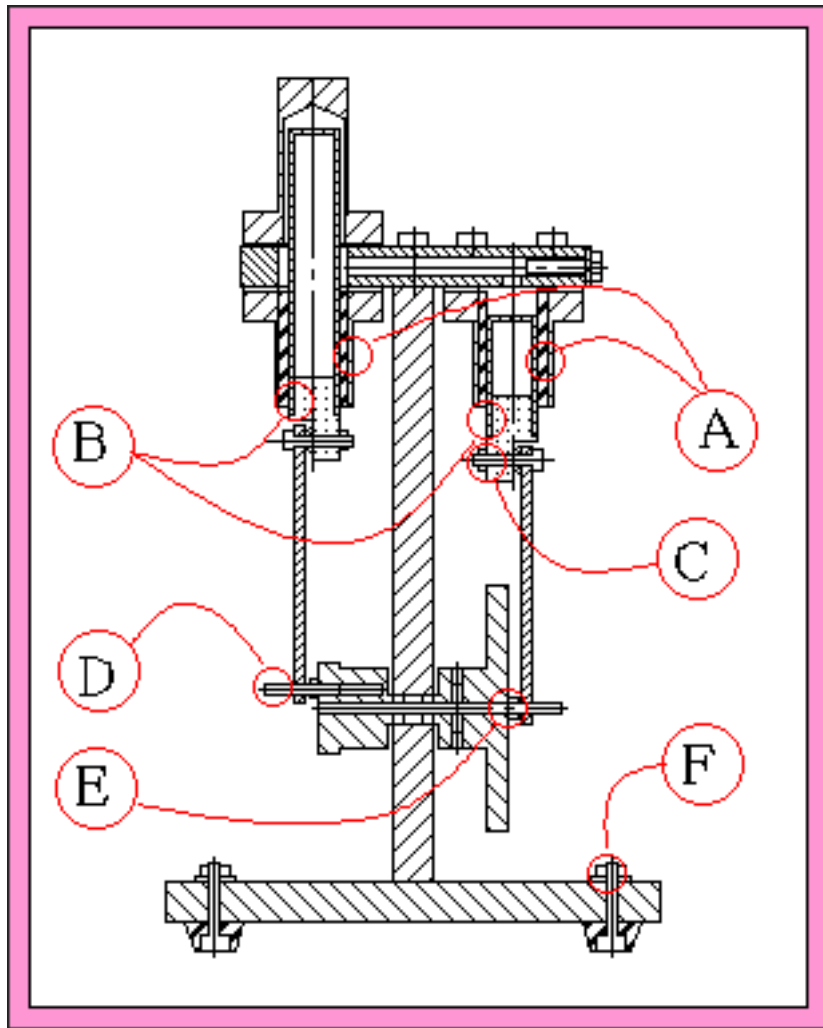


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Suggestions to assemble the engine



A: Seal and fix between a cylinder cover (No.1) and a cylinder (No.6) with a silicone gulue.

B: Fix between a piston holder (No.5) and a piston (No.7,8) with a quick drying glue.

C: Fix a connecting rod (No.13) to a piston holder (No.5) with a bolt (No.24) and a nut (No.26) to move light.

D: Cut a top of a bolt (No.23).

E: Fix the bolts (No.23) to a flywheel (No.3) and a crank disk (No.4) using double nut type.

F: Fix bolts (No.22) to a base (No.11) with double nut type.



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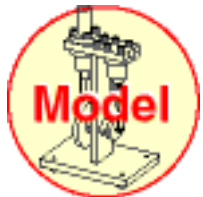
The Stirling engine was invented in 1816, before the gasoline and Diesel engines. The Stirling engine can use any type of fuel, as well as solar energy and hot spring heat. This web site offers academic information for the Stirling engine. Please enjoy these pages.

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[Stirling Engine of Saitama University](#)

The Dynamics of Machinery Laboratory (Prof. Iwamoto) at Saitama University had studied and developed the Stirling engines till March 1998. Here is a web site which was made by Iwamoto's students in these days.



[Building of Model Stirling Engine](#)

Students of the Department of Mechanical Engineering at Saitama University have a class named Mechanical Engineering Seminar. Our laboratory adopted the subject to 'Building of Model Stirling Engine' from September 1997 to January 1998.



[Schmidt Theory for Stirling Engines](#)

This is a famous thermodynamical theory for Stirling engines.



[Simple Performance Prediction Method for Stirling Engine•iJava script•j](#)

I offer a simple method which predicts a output power and engine speed of the Stirling engine. You can calculate on this page.

The web pages entitled 'Study on Design and Performance Prediction Methods for Miniaturized Stirling Engine' and '100 W class Stirling Engine, Ecoboy-SCM81' have been moved to '[Stirling Engine Home Page at National Maritime Research Institute](#)'.



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e-mail: khirata@srilot.go.jp (Office)

SCHMIDT THEORY FOR STIRLING ENGINES

Tentative version on January 20, 1997

KOICHI HIRATA

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1. INTRODUCTION

The Schmidt theory is one of the isothermal calculation methods for Stirling engines. It is the most simple method and very useful during Stirling engine development.

This theory is based on the isothermal expansion and compression of an ideal gas.

2. ASSUMPTION OF SCHMIDT THEORY

The performance of the engine can be calculated a P-V diagram. The volume in the engine is easily calculated by using the internal geometry. When the volume, mass of the working gas and the temperature is decided, the pressure is calculated using an ideal gas method as shown in equation (1).

$$PV = mRT \quad (1)$$

The engine pressure can be calculated under following assumptions:

- (a) There is no pressure loss in the heat-exchangers and there are no internal pressure differences.
- (b) The expansion process and the compression process changes isothermal.
- (c) Conditions of the working gas is changed as an ideal gas.
- (d) There is a perfect regeneration.
- (e) The expansion dead space maintains the expansion gas temperature - T_E , the compression dead space maintains the compression gas temperature - T_C during the cycle.
- (f) The regenerator gas temperature is an average of the expansion gas temperature - T_E and the compression gas temperature - T_C .
- (g) The expansion space - V_E and the compression space - V_C changes according a sine curves.

Table 1 shows symbols used the Schmidt Theory.

Table 1 Symbols

Name	Symbol	Unit
Engine pressure	P	Pa
Swept volume of expansion piston or displacer piston	V_{SE}	m^3
Swept volume of compression piston or power piston	V_{SC}	m^3
Dead volume of expansion space	V_{DE}	m^3
Regenerator volume	V_R	m^3
Dead volume of compression space	V_{DC}	m^3
Expansion space momental volume	V_E	m^3
Compression space momental volume	V_C	m^3
Total momental volume	V	m^3
Total mass of working gas	m	kg
Gas constant	R	J/kgK
Expansion space gas temperature	T_H	K
Compression space gas temperature	T_C	K
Regenerator space gas temperature	T_R	K
Phase angle	dx	deg
Temperatuer ratio	t	
Swept volume ratio	v	
Dead volume ratio	X	
Engine speed	n	Hz
Indicated expansion energy	W_E	J
Indicated compression energy	W_C	J
Indicated energy	W_i	J
Indicated expansion power	L_E	W
Indicated compression power	L_C	W
Indicated power	L_i	W
Indicated efficiency	e	

3. ALPHA-TYPE STIRLING ENGINE

Figure 1 shows the calculation model of Alpha-type Stirling engine.

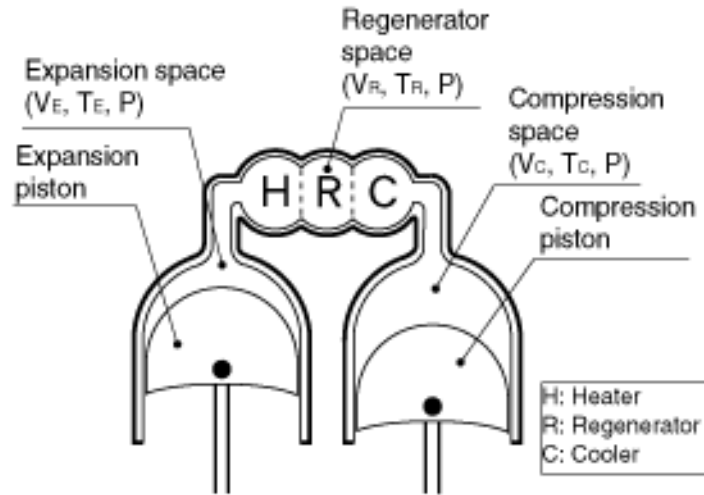


Fig. 1 Alpha-type Stirling Engine

The volumes of the expansion- and compression cylinder at a given crank angle are determined at first. The momental volumes is described with a crank angle - x . This crank angle is defined as $x=0$ when the expansion piston is located the most top position (top dead point).

The momental expansion volume - V_E is described in equation (2) with a swept volume of the expansion piston - V_{SE} , an expansion dead volume - V_{DE} under the condition of assumption (g).

$$V_E = \frac{V_{SE}}{2}(1 - \cos x) + V_{DE} \quad (2)$$

The momental compression volume - V_C is found in equation (3) with a swept volume of the compression piston - V_{SC} , a compression dead volume - V_{DC} and a phase angle - dx .

$$V_C = \frac{V_{SC}}{2}\{1 - \cos(x - dx)\} + V_{DC} \quad (3)$$

The total momental volume is calculated in equation (4).

$$V = V_E + V_R + V_C \quad (4)$$

By the assumptions (a), (b) and (c), the total mass in the engine - m is calculated using the engine pressure - P , each temperature - T , each volume - V and the gas constant - R .

$$m = \frac{PV_E}{RT_E} + \frac{PV_R}{RT_R} + \frac{PV_C}{RT_C} \quad (5)$$

The temperature ratio - t , a swept volume ratio - v and other dead volume ratios are found using the following equations.

$$t = \frac{T_C}{T_E} \quad (6)$$

$$v = \frac{V_{SC}}{V_{SE}} \quad (7)$$

$$X_{DE} = \frac{V_{DE}}{V_{SE}} \quad (8)$$

$$X_{DC} = \frac{V_{DC}}{V_{SE}} \quad (9)$$

$$X_R = \frac{V_R}{V_{SE}} \quad (10)$$

The regenerator temperature - T_R is calculated in equation (11), by using the assumption (f).

$$T_R = \frac{T_E + T_C}{2} \quad (11)$$

When equation (5) is changed using equation (6)-(10), the total gas mass - m is described in the next equation.

$$\square (12)$$

Equation (12) is changed in equation (13), using equation (2) and (3).

$$m = \frac{PV_{SE}}{2RT_C} \{S - B \cos(x - a)\} \quad (13)$$

Now;

$$a = \tan^{-1} \frac{v \cdot \sin dx}{t + \cos dx} \quad (14)$$

$$S = t + 2tX_{DE} + \frac{4tX_R}{1+t} + v + 2X_{DC} \quad (15)$$

$$B = \sqrt{t^2 + 2tv \cos dx + v^2} \quad (16)$$

The engine pressure - P is defined as a next equation using equation (13).

$$P = \frac{2mRT_c}{V_{SE} \{S - B \cos(\theta - a)\}} \quad (17)$$

The mean pressure - P_{mean} can be calculated as follows:

$$P_{mean} = \frac{1}{2\pi} \oint P dx = \frac{2mRT_c}{V_{SE} \sqrt{S^2 - B^2}} \quad (18)$$

c is defined in the next equation.

$$c = \frac{B}{S} \quad (19)$$

As a result, the engine pressure - P, based the mean engine pressure - P_{mean} is calculated in equation (20).

$$P = \frac{P_{mean} \sqrt{S^2 - B^2}}{S - B \cos(x - a)} = \frac{P_{mean} \sqrt{1 - c^2}}{1 - c \cdot \cos(x - a)} \quad (20)$$

On the other hand, in the case of equation (17), when $\cos(x-a)=-1$, the engine pressure - P becomes the minimum pressure - P_{min} , the next equation is introduced.

$$P_{\min} = \frac{2mRT_c}{V_{SE}(S+B)} \quad (21)$$

Therefore, the engine pressure - P, based the minimum pressure - P_{\min} is described in equation (22).

$$P = \frac{P_{\min}(S+B)}{S-B\cos(x-a)} = \frac{P_{\min}(1+c)}{1-c \cdot \cos(x-a)} \quad (22)$$

Similarly, when $\cos(x-a)=1$, the engine pressure - P becomes the maximum pressure - P_{\max} . The following equation is introduced.

$$P = \frac{P_{\max}(S-B)}{S-B\cos(x-a)} = \frac{P_{\max}(1-c)}{1-c \cdot \cos(x-a)} \quad (23)$$

The P-V diagram of Alpha-type Stirling engine can be made with above equations.

4. BETA-TYPE STIRLING ENGINE

Similarly, the equations for Beta-type Stirling engine are declared. Figure 2 shows a calculation model of a Beta-type Stirling engine.

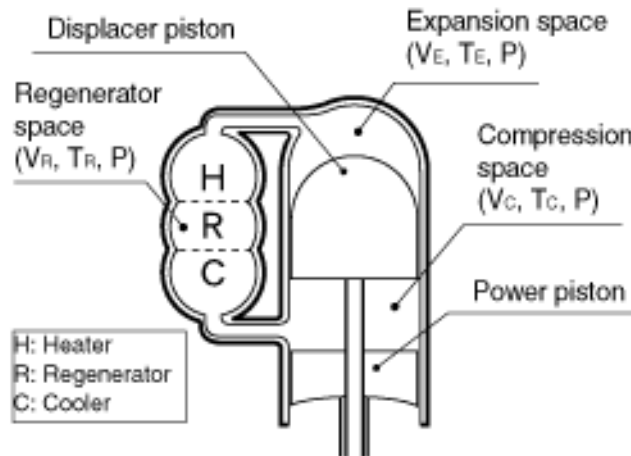


Fig. 2 Beta-type Stirling Engine

The expansion momental volume - V_E and the compression momental volume - V_C are described in the following equations, with a swept volume of a displacer piston - V_{SE} , a swept volume of a power piston -

V_{SC} and a phase angle $-dx$ between the displacer piston and power piston.

$$V_E = \frac{V_{SE}}{2}(1 - \cos x) + V_{DE} \quad (24)$$

$$V_C = \frac{V_{SE}}{2}(1 - \cos x) + \frac{V_{SC}}{2}\{1 - \cos(x - dx)\} + V_{DC} - V_B \quad (25)$$

In the case of the Beta-type Stirling engine, the displacer piston and the power piston are located in the same cylinder. When both pistons overlaps there stroke, an effective working space is created. The overlap volume - V_B in equation (25) can be calculated in the next equation.

$$V_B = \frac{V_{SE} + V_{SC}}{2} - \sqrt{\frac{V_{SE}^2 + V_{SC}^2}{4} - \frac{V_{SE} V_{SC}}{2} \cos dx} \quad (26)$$

Then the total momental volume - V is found in equation (27).

$$V = V_E + V_R + V_C \quad (27)$$

The engine pressure - P based the mean pressure - P_{mean} , the minimum pressure - P_{min} and the maximum pressure - P_{max} are described in the following equations like the Alpha-type Stirling engine.

$$P = \frac{P_{mean} \sqrt{1 - c^2}}{1 - c \cdot \cos(x - a)} = \frac{P_{min} (1 + c)}{1 - c \cdot \cos(x - a)} = \frac{P_{max} (1 - c)}{1 - c \cdot \cos(x - a)} \quad (28)$$

Several ratios and coefficients are defined as follows.

$$t = \frac{T_C}{T_E} \quad (29)$$

$$v = \frac{V_{SC}}{V_{SE}} \quad (30)$$

$$X_B = \frac{V_B}{V_{SE}} \quad (31)$$

$$X_{DE} = \frac{V_{DE}}{V_{SE}} \quad (32)$$

$$X_{DC} = \frac{V_{DC}}{V_{SE}} \quad (33)$$

$$X_R = \frac{V_R}{V_{SE}} \quad (34)$$

$$a = \tan^{-1} \frac{v \sin dx}{t + \cos dx + 1} \quad (35)$$

$$S = t + 2tX_{DE} + \frac{4tX_R}{1+t} + v + 2X_{DC} + 1 - 2X_B \quad (36)$$

$$B = \sqrt{t^2 + 2(t-1)v \cos dx + v^2 - 2t + 1} \quad (37)$$

$$c = \frac{B}{S} \quad (38)$$

The P-V diagram of Beta-type Stirling engine can be made with above equations.

5. GAMMA-TYPE STIRLING ENGINE

Figure 3 shows a calculation model of a Gamma-type Stirling engine.

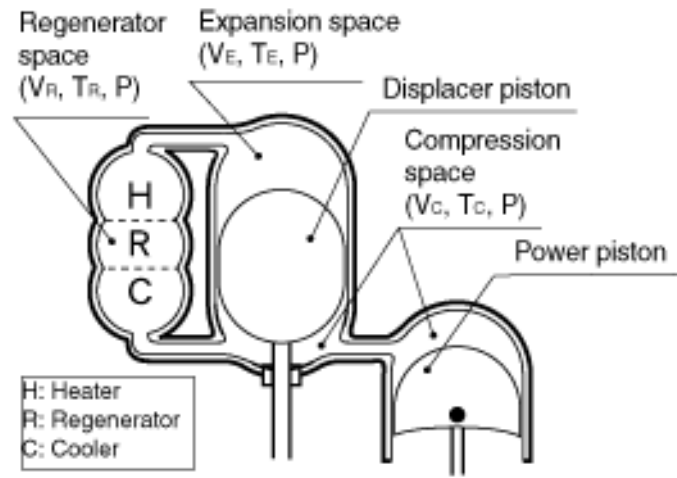


Fig. 3 Gamma-type Stirling Engine

Similar calculation equations are made as the Alpha- and Beta-type engine. The expansion momental volume - V_E and the compression momental volume - V_C are described in the following equations with a swept volume of a displacer piston - V_{SE} , a swept volume of a power piston - V_{SC} and a phase angle - dx between the displacer piston and the power piston.

$$V_E = \frac{V_{SE}}{2}(1 - \cos x) + V_{DE} \quad (39)$$

$$V_C = \frac{V_{SE}}{2}(1 - \cos x) + \frac{V_{SC}}{2}\{1 - \cos(x - dx)\} + V_{DC} \quad (40)$$

The total momental volume - V is described a next equation.

$$V = V_E + V_R + V_C \quad (41)$$

The engine pressure - P based the mean pressure - P_{mean} , the minimum pressure - P_{min} and the maximum pressure - P_{max} are found in the following equations.

$$P = \frac{P_{mean} \sqrt{1 - c^2}}{1 - c \cdot \cos(x - a)} = \frac{P_{min} (1 + c)}{1 - c \cdot \cos(x - a)} = \frac{P_{max} (1 - c)}{1 - c \cdot \cos(x - a)} \quad (42)?$$

Now,

$$t = \frac{T_C}{T_E} \quad (43)$$

$$v = \frac{V_{SC}}{V_{SE}} \quad (44)$$

$$X_{DE} = \frac{V_{DE}}{V_{SE}} \quad (45)$$

$$X_{DC} = \frac{V_{DC}}{V_{SE}} \quad (46)$$

$$X_R = \frac{V_R}{V_{SE}} \quad (47)$$

$$a = \tan^{-1} \frac{v \sin dx}{t + \cos dx + 1} \quad (48)$$

$$S = t + 2tX_{DE} + \frac{4tV_R}{1+t} + v + 2X_{DC} + 1 \quad (49)$$

$$B = \sqrt{t^2 + 2(t-1)v \cos dx + v^2 - 2t + 1} \quad (50)$$

$$c = \frac{B}{S} \quad (51)$$

The P-V diagram of Gamma-type Stirling engine can be made with above equations.

6. INDICATED ENERGY, POWER AND EFFICIENCY

The indicated energy (area of the P-V diagram) in the expansion and compression space can be calculated as an analytical solutions with use of the above coefficients. The indicated energy in the expansion space (indicated expansion energy) - W_E (J), based on the mean pressure - P_{mean} , the minimum pressure - P_{min} and

the maximum pressure - P_{\max} are described in the following equations.

$$W_E = \oint P dV_E = \frac{P_{\text{mean}} V_{SE} \pi c \cdot \sin a}{1 + \sqrt{1 - c^2}} = \frac{P_{\min} V_{SE} \pi c \cdot \sin a}{1 + \sqrt{1 - c^2}} \cdot \frac{\sqrt{1 + c}}{\sqrt{1 - c}} = \frac{P_{\max} V_{SE} \pi c \cdot \sin a}{1 + \sqrt{1 - c^2}} \cdot \frac{\sqrt{1 - c}}{\sqrt{1 + c}} \quad (52)$$

The indicated energy in the compression space (indicated compression energy) - W_C (J) are described in the next equations.

$$W_C = \oint P dV_C = -\frac{P_{\text{mean}} V_{SE} \pi c t \cdot \sin a}{1 + \sqrt{1 - c^2}} = -\frac{P_{\min} V_{SE} \pi c t \cdot \sin a}{1 + \sqrt{1 - c^2}} \cdot \frac{\sqrt{1 + c}}{\sqrt{1 - c}} = -\frac{P_{\max} V_{SE} \pi c t \cdot \sin a}{1 + \sqrt{1 - c^2}} \cdot \frac{\sqrt{1 - c}}{\sqrt{1 + c}} \quad (53)$$

The indicated energy per one cycle of this engine - W_i (J) is demanded in the next equations.

$$\begin{aligned} W_i &= W_e + W_c \\ &= \frac{P_{\text{mean}} V_{SE} \pi c (1 - t) \sin a}{1 + \sqrt{1 - c^2}} = \frac{P_{\min} V_{SE} \pi c (1 - t) \sin a}{1 + \sqrt{1 - c^2}} \cdot \frac{\sqrt{1 + c}}{\sqrt{1 - c}} = \frac{P_{\max} V_{SE} \pi c (1 - t) \sin a}{1 + \sqrt{1 - c^2}} \cdot \frac{\sqrt{1 - c}}{\sqrt{1 + c}} \quad (54) \end{aligned}$$

Relations between P_{mean} , P_{\min} and P_{\max} are determined in the following equations.

$$\frac{P_{\min}}{P_{\text{mean}}} = \sqrt{\frac{1 - c}{1 + c}} \quad (55)$$

$$\frac{P_{\max}}{P_{\text{mean}}} = \sqrt{\frac{1 + c}{1 - c}} \quad (56)$$

The indicated expansion power - L_E (W), the indicated compression power - L_C (W) and the indicated power of this engine - L_i (W) are defined in the following equations, using the engine speed per one second, n (rps, Hz).

$$L_E = W_E n \quad (57)$$

$$L_c = W_c n \quad (58)$$

$$L_i = W_i n \quad (59)$$

The indicated expansion energy - W_E found equation (52) means an input heat from a heat source to the engine. The indicated compression energy - W_c calculated by equation (53) means a reject heat from the engine to cooling water or air. Then the thermal efficiency of the engine - e is calculated in the next equation.

$$e = \frac{W_i}{W_E} = 1 - t \quad (60)$$

This efficiency equals that of a Carnot cycle which is the most highest efficiency in every thermal engine.

7. EXAMPLE OF CALCULATION

EXERCISE:

Make a P-V diagram and calculate the indicated power of an Alpha-type Stirling engine under following conditions.

Swept volume of an expansion piston: 0.628 cm^3 , swept volume of a compression piston: 0.628 cm^3 , dead volume of the expansion space: 0.2 cm^3 , dead volume of the compression space: 0.2 cm^3 , regenerator volume: 0.2 cm^3 , phase angle: 90 deg , mean pressure: 101.3 kPa , expansion gas temperature: 400 degC , compression gas temperature: 30 degC , engine speed: 2000 rpm .

A temperature ratio - t , a swept volume ratio - v and other dead volume ratio are calculated with the equation (6) - (10).

$$t = \frac{30 + 273}{400 + 273} = 0.450$$

$$v = \frac{0.628 \times 10^{-6}}{0.628 \times 10^{-6}} = 1.000$$

$$X_{DE} = \frac{0.2 \times 10^{-6}}{0.628 \times 10^{-6}} = 0.318$$

$$X_{DC} = \frac{0.2 \times 10^{-6}}{0.628 \times 10^{-6}} = 0.318$$

$$X_R = \frac{0.2 \times 10^{-6}}{0.628 \times 10^{-6}} = 0.318$$

Each coefficient is calculated with equation (14) - (16) and (19).

$$a = \tan^{-1} \frac{1 \times \sin 90^\circ}{0.45 + \cos 90^\circ} = 65.772^\circ$$

$$S = 0.45 + 2 \times 0.45 \times 0.318 + \frac{4 \times 0.45 \times 0.318}{1 + 0.450} + 1 + 2 \times 0.318 = 2.767$$

$$B = \sqrt{0.45^2 + 2 \times 0.45 \times \cos \frac{\pi}{2} + 1} = 1.097$$

$$c = \frac{1.097}{2.767} = 0.396$$

Engine pressure is calculated with equation (20).

When crank angle - $x=0\text{deg}$:

$$P = \frac{101.3 \times 10^3 \sqrt{1 - 0.396^2}}{1 - 0.396 \cos(0 - 65.772)} = 101.988 \times 10^3 \text{ (Pa)} = 101.988 \text{ (kPa)}$$

Similarly, when $x=10\text{deg}$:

$$P = 109.893 \text{ (kPa)}$$

When $x=20\text{deg}$:

$$P = 118.011 \text{ (kPa)}$$

•c

Next each momental volume is calculated with equation (2) - (4).

When crank angle, $x=0\text{deg}$:

$$V_E = \frac{0.628 \times 10^{-6}}{2} (1 - \cos 0^\circ) + 0.2 = 0.200 \times 10^{-6} (\text{m}^3) = 0.200 (\text{cm}^3)$$

$$V_C = \frac{0.628 \times 10^{-6}}{2} \{1 - \cos(0^\circ - 90^\circ)\} + 0.2 = 0.514 \times 10^{-6} (\text{m}^3) = 0.514 (\text{cm}^3)$$

$$V = 0.2 + 0.2 + 0.514 = 0.914 (\text{cm}^3)$$

When $x=10\text{deg}$:

$$V = 0.864 (\text{cm}^3)$$

When $x=20\text{deg}$:

$$V = 0.826 (\text{cm}^3)$$

•c

Repeat above calculation to one complete cycle and plot the volumes - V and pressures - P on a graph paper. An example of the P-V diagram is shown in Fig. 4.

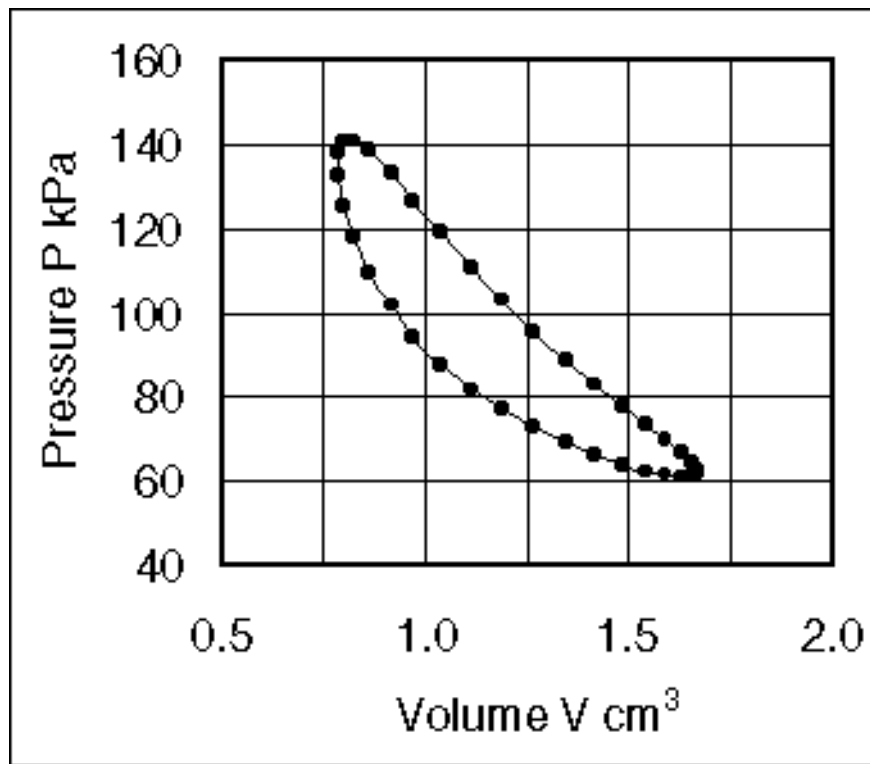


Fig. 4 P-V Diagram

The indicated energy is calculated with equation (52), (53) and (54).

$$W_{\mathbf{r}} = \frac{101.3 \times 10^3 \times 0.628 \times 10^{-4} \times 3.14 \times 0.396 \times \sin 65.772^\circ}{1 + \sqrt{1 - 0.396^2}} = 3.760 \times 10^{-2} (\text{J})$$

$$W_{\mathbf{c}} = -\frac{101.3 \times 10^3 \times 0.628 \times 10^{-4} \times 3.14 \times 0.396 \times 0.45 \times \sin 65.772^\circ}{1 + \sqrt{1 - 0.396^2}} = -1.692 \times 10^{-2} (\text{J})$$

$$W_{\mathbf{i}} = 3.760 \times 10^{-2} - 1.692 \times 10^{-2} = 2.068 \times 10^{-2} (\text{J})$$

The indicated power of this engines is calculated with equation (59).

$$L_{\mathbf{i}} = \frac{5.452 \times 10^{-2} \times 2000}{60} = 0.689 (\text{W})$$

The indicated power of this engine is 0.689 W.

ACKNOWLEDGEMENT

I wish to thank J. H. de Baat for many helpful suggestions during the translation of this script.

REFERENCES

- 1) G. Walker., Stirling Engines, (1980),17, Oxford Univ. Press.
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Simple Performance Prediction Method for Stirling Engine

Simple Performance Prediction Method for Stirling Engine ver. 1.8J
Koichi Hirata (31 March, 1998)

[Japanese page](#)

How to use this method

- (1) Please input or select a mean pressure, a swept volume, gas temperatures and a kind of the working gas on the calculated condition.
- (2) When the engine is operated at the design pressure and the permitted temperature (maximum temperature), please check '**Use the design and permitted values**'.
- (3) If not so, please check '**Do not use the design and permitted values**' and input the design pressure and the permitted temperature.
- (4) Please click 'START'.
- (5) Maximum output power and the engine speed when the engine get the maximum output power are calculated under my experimental equations.

Calculated condition

Mean pressure, P_m : (MPa)

Swept volume of expansion space, V_{se} : (cm³)

Gas temperature of expansion space, T_e : (deg C)

Gas temperature of compression space, T_c : (deg C)

Working gas,

He

Air

N₂

H₂

Use the design and permitted values (You need not input following values.)

Do not use the design and permitted values (You must input following values.)

Design pressure, P_{lim} : (MPa)

Permitted temperature, T_{lim} : (°C)

<-- Click here after input all calculated conditions!

Calculated results

Viscosity coefficient, ν_{lim} : (m²/s)

Gas constant, R: (J/kgK)

Non-dimensional engine specification, S^* :

Non-dimensional output power, L_s^* :

Non-dimensional engine speed, n^* :

Maximum output power, L_s : (W)

Engine speed, N: (rpm)

Beale number, B_n :

West number, W_n :

* In the case of the high temperature difference Stirling engines, a standard value of Beal number is 0.15.

* A standard value of West number is from 0.25 to 0.35.

Additional information



[Outline of this calculated method](#)



[Difference from former methods](#)



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Outline of this calculated method

Following non-dimensional numbers is used in this prediction method.

$$n^* \equiv \frac{n V_{SE}^{2/3}}{\nu}$$

Non-dimensional engine speed, n^* :

$$W_s^* \equiv \frac{W_s}{P_m V_{SE} P^* T^*}$$

Non-dimensional work, W_s^* :

Non-dimensional output power, L_s^* : $L_s^* = W_s^* \cdot n^*$

$$S^* = \frac{T_{lim} R V_{SE}^{2/3}}{\nu_{lim}^2}$$

Non-dimensional engine specification, S^* :

$$P^* \equiv \frac{P_m}{P_{lim}}$$

Design pressure ratio, P^* :

$$T^* \equiv \frac{T_E - T_C}{T_E + T_C}$$

Non-dimensional temperature, T^* :

W_s : Output work in one cycle (J)

P_m : Mean pressure (Pa)

P_{lim} : Design pressure (Pa)

V_{SE} : Swept volume of expansion space (m³)

R: Gas constant (J/kgK)

T_E : Gas temperature of expansion space (K)

T_C : Gas temperature of compression space (K)

T_{lim} : Maximum temperature in design process (K)

ν : Viscosity coefficient at temperature, T_E and pressure, P_m (m²/s)

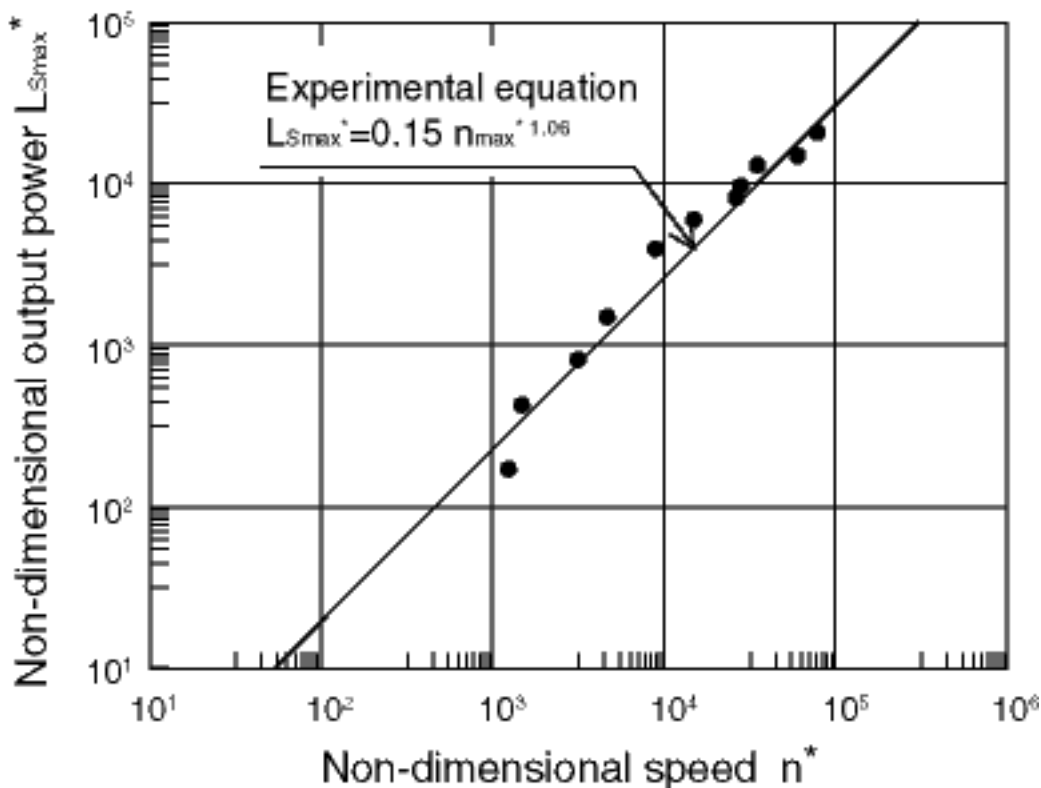
ν_{lim} : Viscosity coefficient at temperature, T_{lim} and pressure, P_{lim} (m²/s)

n: Engine speed (rps)

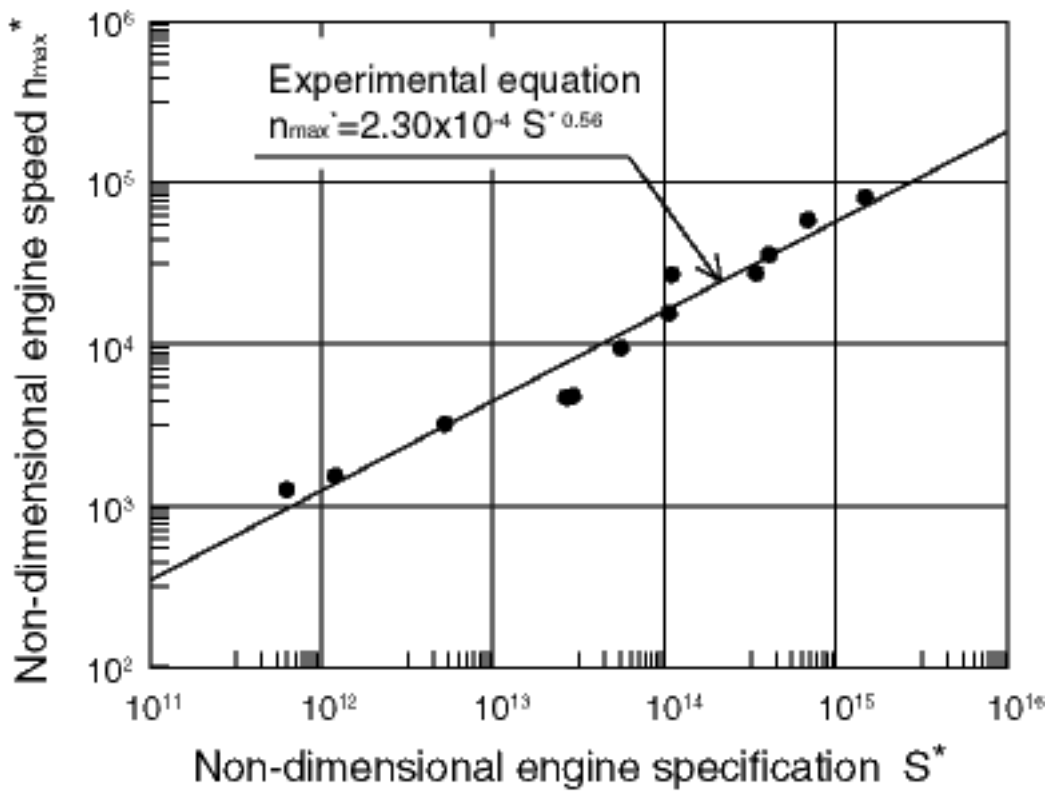
* When you check 'Use the design and permitted values', $P_{lim}=P_m$ and $T_{lim}=T_E$.

I have reduced the experimental data of Ecoboy-SCM81 and other high performance and low temperature difference Stirling engines.

Following figures show the relation between the non-dimensional engine speed, n_{max}^* , and the non-dimensional output power, L_{Smax}^* , and the relation between the non-dimensional engine specification, S^* , and the non-dimensional engine speed, n_{max}^* . Experimental equations which are shown in the figures are estimating the characteristics of the engines well.



Non-dimensional output power, L_{Smax}^* as a function of non-dimensional engine speed, n_{max}^*



Non-dimentional engine speed, n_{max}^* as a function of non-dimentional non-dimentional engine specification, S^*

This performance prediction method calculates under above experimental equations.

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Difference from former methods

As the former simple performance prediction methods for the Stirling engines, Beale number, B_N , or West number, W_N is used.

$$B_N = \frac{L_s}{P_m V_{SE} n}$$

$$W_N = \frac{L_s}{P_m V_{SE} n \left(\frac{T_E - T_C}{T_E + T_C} \right)}$$

P_m : Mean pressure (Pa)

V_{SE} : Swept volume of expansion space (m³)

n : Engine speed (rps)

T_E : Gas temperature of expansion space (K)

T_C : Gas temperature of compression space (K)

In the case of the high temperature difference Stirling engines, a standard value of Beal number is 0.15. A standard value of West number is from 0.25 to 0.35.

The output power can be calculated approximately, when these numbers are used. But, it is necessary that the engine speed when the engine gets the maximum output power is supposed.

In the case of the simple performance prediction method which is proposed on my web page, the output power and the engine speed can be predicted easily. And effects of the kind of the working gas is estimated when the non-dimentional engine specification is introduced.

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How does a Stirling engine work?

[To Japanese Page](#)

Step 1 Characteristics of air

Let's fit a rubber to a can like Figure 1. You can understand easily that the rubber expands when the can is heated (Figure 2), and the rubber contracts when the can is cooled (Figure 3). It is caused that a pressure of the air in the can works to the rubber when the air is heated, shown in all of Figure 2. Of course, you cannot see the pressure by your eyes.

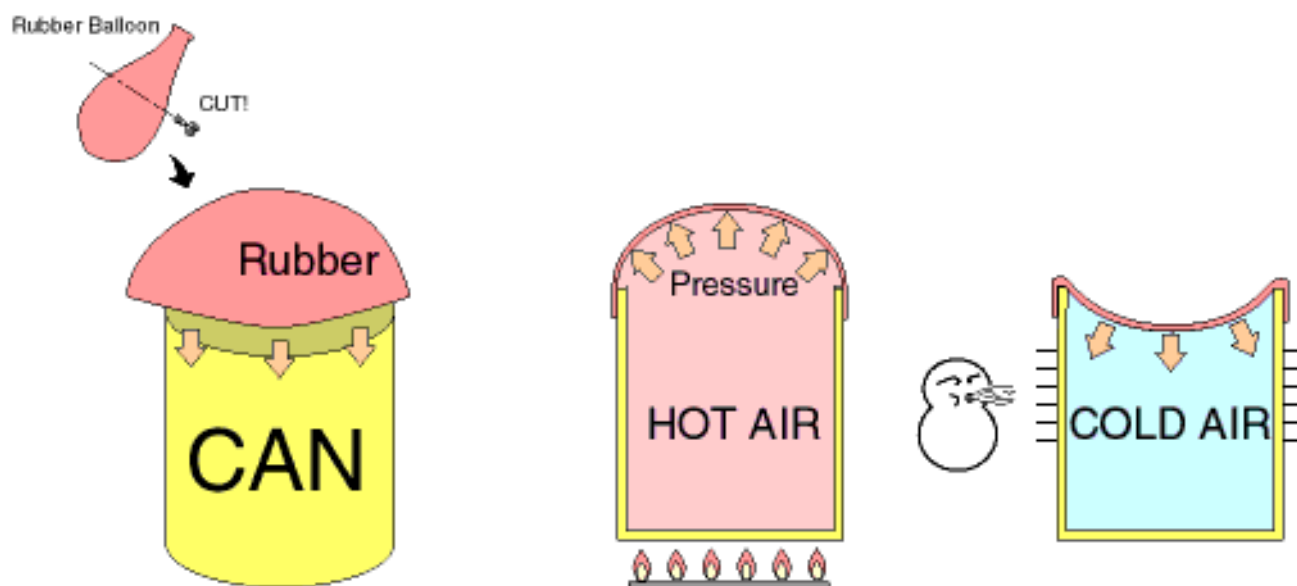


Fig. 1 Can and rubber

Fig. 2 Air is heated...

Fig.3 Air is cooled...

Step 2 What is a displacer piston?

Next, let's put in a piston into the can like Figure 4. A diameter of the piston must be somewhat smaller than that of the can, because the piston works to move (displace) the air up and down in the can. And, please heat the bottom side of the can and cool the upper side of the can. After it has enough temperature difference, move the piston up and down by your hand. When the piston is moved up, the rubber expands because there is a lot of hot air in the can (Figure 5). It corresponds to Figure 2. When the piston is moved down, the rubber contracts because there is a lot of cold air in the can. It corresponds to Figure 3.

In the case of the Stirling engine, this piston, which moves (displaces) the air and make the pressure changes is called a displacer piston.

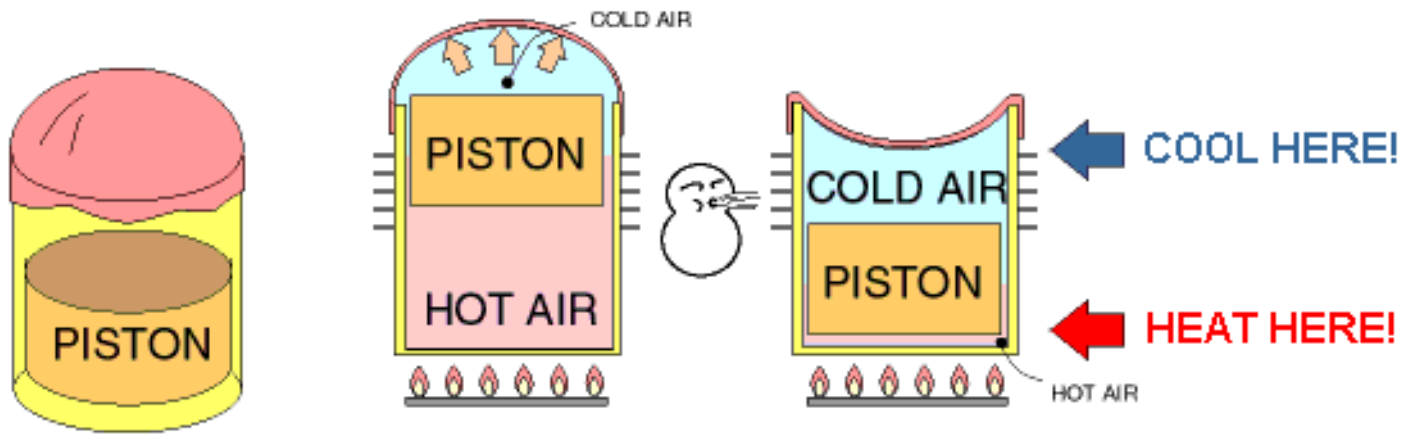


Fig. 4 Displacer piston

Fig. 5 Work of piston

Step 3 Crank mechanism ~ from piston motion to rotation ~

Do you understand about the characteristics of the air and the work of the displacer piston? They are very important to understand how does the Stirling engine work.

First, connect from the piston and a bent wire with a thread like Figure 6. When the bent wire is rotated, the piston is moved up and down. This is called a crank mechanism.

Please heat the bottom side of the can and cool the upper side of the can, similar to above description. When your hand rotates the bent wire, the piston is moved up and down, and the rubber expands and contracts repeatedly (Figure 7).

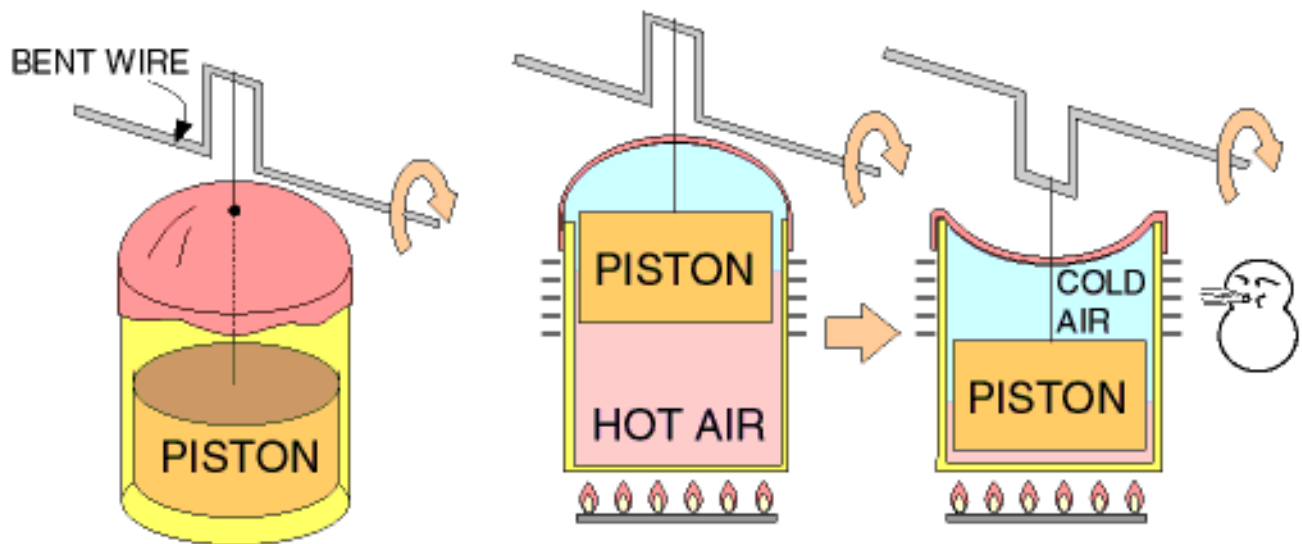


Fig. 6 Crank mechanism

Fig. 7 Work of crank mechanism

Step 4 Power piston ~ function of the rubber ~

The Stirling engine converts from the motion of the rubber to the rotation of the bent wire. Please connect from the rubber to the bent wire with a rod. In this time, a force of the rubber (expansion and contraction) has to be the direction, which rotates the bent wire. In short, you must bend the bent wire the just right angle (90

degrees) from the piston like Figure 8 and 9.

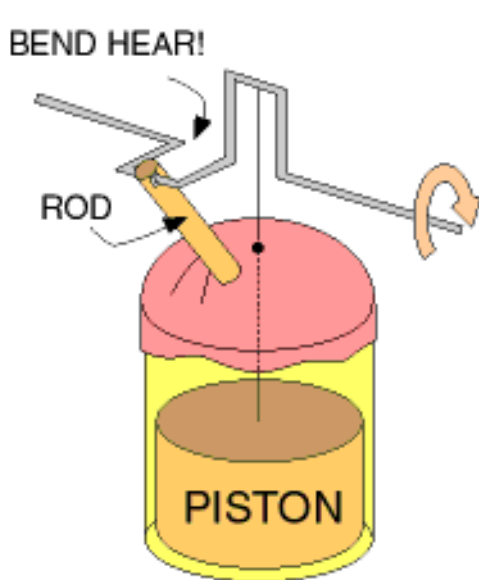


Fig. 8 Force of rubber

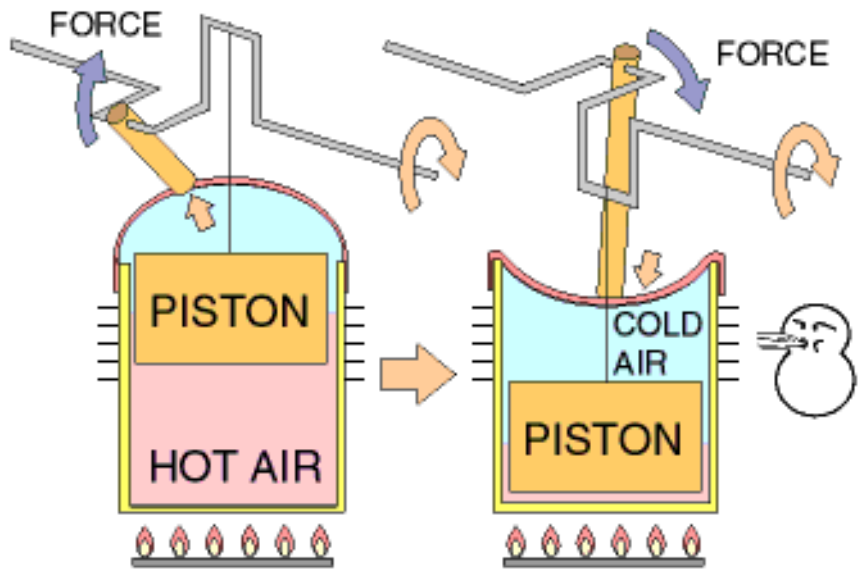


Fig. 9 Force for rotation

Step 5 Flywheel ~ for the smooth rotation ~

This engine has not worked yet. If you try to work this engine (Figure 8 and 9), the rubber keeps the most expansion or contraction. In order to rotate smoothly and repeatedly, you must fit a rotated mass with the bent wire. It is called a flywheel.

Generally, the flywheel is circular like Figure 10. But at this point, please bend the end of the bent wire, and fit a mass at the edge of the wire like Figure 11. The mass works as the flywheel and to be balanced to the weight of the piston. So, you must fit the mass against the piston.

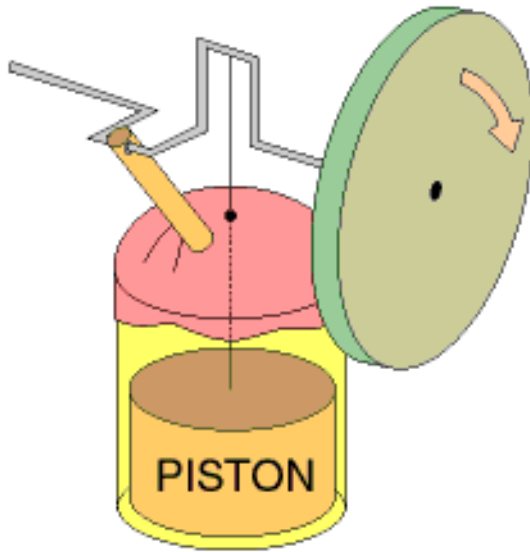


Fig. 10 Circular type Flywheel

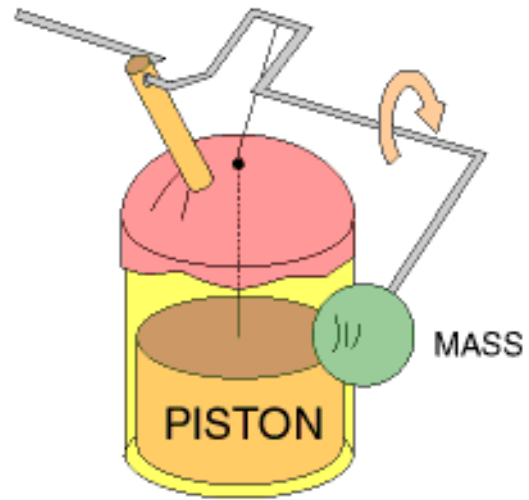


Fig. 11 Simple type Flywheel

Step 6 It is the Stirling engine

Finally, the Stirling engine is completed.

Do you understand how does the Stirling engine work?

Yes.

No.

I have known the principles before seeing this page.

Do you think that above description is easy to understand?

Yes, it is easy.

No, it is not easy.

If you have any suggestion about this page, please tell me it.

If you can, please tell me your name, e-mail address and country.

Name:

Country:

E-mail address:

P.S.: When the Stirling engine is understood, it is important that the knowledge of the characteristics of air (*Thermodynamics*), the crank mechanism and the flywheel (*Mechanics and Mechanical Vibrations*). I have tried to explain about them as simply as possible. Please contact me if you find incomprehensible contents or any errors, and have any your opinions. I hope that this description becomes more suitable for every young student. Permission is hereby granted, by author, to print and distribute this description. Thank you.

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